Foundations of Finance Theory, Exam
January, 2006

Answer all questions

question 1 (25 points)
question 2 (35 points)
question 3 (25 points)
question 4 (15 points)

1. (25 marks)

   (a) Assume that the forward price can be written as

   $$F_j = E\left[\phi(x_m)\ x_j\right].$$

   Explain why this can be written as

   $$F_j = E\left(x_j\right) + cov\left[\phi(x_m), x_j\right],$$

   (b) Explain when and why the forward price may be written as

   $$F_j = E\left(x_j\right) - \kappa cov\left(x_j, x_m\right),$$

   where κ is a constant.

   (c) Explain why κ in b) may be expressed as

   $$\kappa = \frac{E(x_m) - F_m}{var(x_m)},$$

   (d) Given the model above, give an expression for the rate of return

   $$\frac{E(x_j) - F_j}{F_j}$$

   (e) Assuming the following data: $r_f = 5\%$, $\beta_j = 1.3$, $E(r_m) = 8\%$, compute the cost of capital for company j

2. (35 marks)

   (a) Assume that the forward price of an option is given by

   $$F[g(x_j)] = E[g(x_j)\phi(x_m)].$$

   Explain why this can be re-written as

   $$F[g(x_j)] = E[g(x_j)\psi(x_j)]$$

   and give an expression for $\psi(x_j)$.

   (b) What is $E[\psi(x_j)]$ and why is it this value?

   (c) Assume that $\psi(x_j)$ has constant elasticity and hence can be written as

   $$\psi(x_j) = \alpha x_j^{\beta}$$

   Also, assume that x_j is lognormal. Give an expression for $\psi(x_j)$.

   (d) Given $\psi(x_j)$, state the risk-adjusted probability density of x_j

   (e) Assuming that x_j and $\psi(x_j)$ are joint-lognormal what is the forward price of x_j?

   (f) Using the forward price, F_j, give an expression for the forward price of a call option which is a risk-neutral valuation relationship.

   (g) State the Black model for the forward price of a call option on x_j

   (h) For a stock which pays a known dividend D_{t+T} at time $t+T$, give the spot-forward parity relationship.

   (i) Give the formula for the value of a call option on such a stock in h) above

3. (25 marks)

   (a) From Cox, Ingersoll and Ross (1981), the futures price of x_{t+2} is the value of a contract paying what, at $t+2$?

   (b) The futures price is given by

   $$H_{t,t+2} = E_t(x_{t+2}),$$

   State whether this is True or False

   (c) For futures and forwards on bonds and interest rates

   i. Forward price > futures price
   ii. Forward rate > futures rate
   iii. Forward price = futures price

   Choose one of the following options:

   a) i. is correct
   b) ii. is correct
   c) iii. is correct
   d) i. and ii. are correct
   f) ii. and iii. are correct

   (d) For this question, assume that x_{t+T}, b_{t+T}, and ϕ_{t+T} are lognormal variables. Let $cov(\ln x, \ln b) = 0.1$. What is $H_{t,t+T}/F_{t,t+T}$?

   (e) Assume that interest rates follow a one-factor model and the futures and forward rates are defined by

   $$H_{t,t+T} = e^{-h_{t,t+T}}$$

   and

   $$F_{t,t+T} = e^{-f_{t,t+T}}$$

   respectively. Let the covariance between the 4-year bond and the discount factor be

   $$\sigma_{xb} = \frac{4(4-1)}{2}(0.01)^2.$$

   Compute the forward rate assuming that the futures rate is 5.5%.

4. (15 marks)

   Let $\phi_{t,t+1} = q_i/p_i$, $\phi_{t,t+2} = q_k/p_k$, $\phi_{t+1,t+2} = q_{i,k}/p_{i,k}$.

   Which of the following statements are true and which are false. In each case give a brief explanation of your answer.

   In a complete market:

   (a)

   $$\phi_{t,t+2} = \phi_{t,t+1}\phi_{t+1,t+2}$$

   (b)

   $$S_{j,t} = B_{t,t+1}E_t(\phi_{t,t+1}S_{j,t+1})$$

   (c)

   $$S_{j,t} = B_{t,t+1}[E_t(S_{j,t+1}) + \lambda cov(S_{j,t+1}, S_{m,t+1})]$$